## UNL 2201: Space, Time & Matter **Tutorial 9**

1) The 'Doppler Effect' is responsible for the siren of an approaching fire engine having a higher pitch or frequency as perceived by a stationary observer, and correspondingly, a receding fire engine siren to have a lower frequency (as perceived by a stationary observer). Is it true or false that the Doppler frequency shift produced by you receding from the source of sound is the same as the shift produced by the source of sound receding from you?

2) The special relativistic formula for the Doppler effect (for either light waves or pulses) is:

$$\lambda' = \lambda \frac{\sqrt{(1+ v/c)}}{\sqrt{(1- v/c)}}$$

where λ is the emitted wavelength as measured in the source's reference frame, and λ' is the wavelength measured in a frame moving with speed v away from the source along the line of sight *i.e.* v is the relative velocity between source and detector. (For relative motion toward each other, v<0 in this formula.) Obtain the above formula using spacetime diagrams.

For speeds much smaller than the speed of light, *i.e.* v<<c, show that this formula reduces to

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta\lambda}{\lambda} = \pm \frac{v}{c}$$

3) Let's consider again the spaceship (Tutorial 6) traveling between two planets, A and B and emitting flashes of light every 6 minutes. As in Tutorial 6, it travels away from A and towards B. If its flashes are seen at 3 minute intervals on B, use the Doppler effect to determine that on planet A, the flashes are seen at:

(a) 3 minute (b) 6 minute (c) 9 minute (d) 12 minute intervals.

4) A comet is chasing a spacecraft. Let v be the speed of the comet, p its momentum and E its energy, all as perceived by the astronaut when the comet hits the spacecraft. In what way would increasing the spacecraft's speed alter the astronaut's perceived values of v, p and E?

(a) v, p, E will all not change

(b) v, p, E will all decrease

(c) v, p will get smaller but E will not change

(d) v and E will decrease but p will not change

(e) E and p will decrease while v will not change.

5) The same spacecraft as in the previous question is now being chased by a powerful laser beam. Let v, p and E be the velocity, momentum and energy of a photon in the laser beam as perceived by the astronaut when it hits the spacecraft. As in the previous question, in what way will increasing the spacecraft's speed alter the astronaut's perceived values of v, p and E? Choose one of (a) – (e) given in the previous question 4).

6) The time dilation effect in Special Relativity means moving clocks run *slower* than 'stationary' clocks. On the other hand, if we consider two stationary clocks, one of which is at a higher altitude above the earth's surface than the other, Einstein's Principle of Equivalence implies that the clock that is placed at the higher elevation will run *faster* compared to the clock which remains on the ground.

In terms of frequency, a clock moving at a velocity v will register a frequency

$$f' = f \sqrt{(1 - v^2/c^2)}$$

where f is the frequency of the 'stationary' clock; while a clock placed at a height H above another clock of frequency f will register a frequency

$$f' = f\,(1 + g\,H/c^2).$$

Let's say we have two identical clocks, *A* and *B*, sitting together on the surface of the earth. Now, we lift clock *A* vertically to some height H, hold it there awhile, and return it to the ground so that it arrives just at the instant when clock *B* has advanced by 100 seconds. How *should we move clock A so that it reads the latest possible time, but always assuming that it* returns when *B* reads exactly 100 seconds?

Next, consider a slightly different problem; say we have two points *A* and *B* both on the earth's surface at some distance from one another. We ask how we should go from *A* to *B* so that the time on our moving watch (*i.e.* it's *proper time*) will be the longest — assuming we start at *A* on a given signal and arrive at *B* on another signal at *B* which we will say is 100 seconds later as measured by a 'stationary' clock.

Assuming that the speed of the moving clock is much less than the speed of light, show that the net frequency shift of the moving clock is

$$\Delta f \;=\; \frac{f}{c^2}\,(g\,H - v^2/2)$$

and hence that the time excess registered by the moving clock over the entire trajectory from *A* to *B* will be:

$$\frac{1}{c^2}\int (g\,H - v^2/2)\,dt$$

which is supposed to be a *maximum.*

The above reasoning leads us to conclude that Newton's law of motion as it applies to a freely falling object in a *uniform* gravitational field can instead be restated as the Einsteinian law: *an object always moves from one place to another so that a clock carried on it gives a longer proper time than it would on any other possible trajectory* (with, of course, the same starting and finishing conditions). In other words, **in free fall, the trajectory makes the proper time of the falling object a maximum.**

The resulting motion is what would be obtained by applying Newton's second law because the latter can be restated in terms of the *principle of least action**, which in the present case requires that the object moves from *A* to *B* in such a way that the quantity

$$\int (\tfrac{1}{2}\,m_o v^2 \;-\; m_o\,g\,H)\,dt \qquad \text{is a } \underline{\textit{minimum.}}$$

* see, for instance, Chapter 19, Feynman Lectures in Physics Vol II

7) Show that in Newton's theory of gravity, a light ray that passes by the Sun at a distance $r_0$ from its centre will, according to Newton's theory, be deflected by an amount $\delta_{Newton}=2GM/c^2 r_0$ where M is the mass of the Sun.

For a light ray that just grazes the surface of the Sun, $r_0$ will equal the Sun's radius. Upon substituting appropriate values for the quantities in the deflection equation verify that in this case, $\delta_{Newton} = 0.875''$.

*Hint: Assume that the 'light particle' travels along a nearly straight line path (while this is a rather crude approximation, it nevertheless correctly gives the Newtonian deflection to leading order).*