UNL 2201: Space, Time & Matter **Tutorial 8**

1. Suppose that both a bowling ball and a feather fall vertically from a height of 100 metres. As they fall to the ground, which encounters the greatest force of air resistance?

a) the bowling ball b) the feather c) both the same

2. What will happen if two identical cannons are aimed at each other and the shells fired simultaneously and at the same speeds? One cannon is higher than the other, but the two are perfectly aligned.

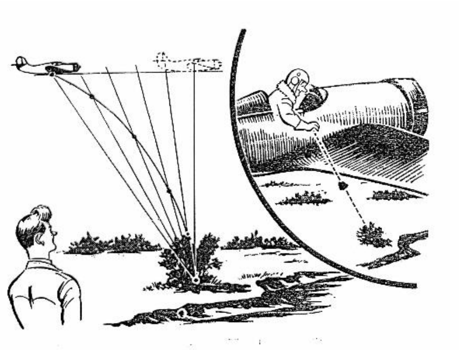
3. A boulder is many times heavier than a pebble – that is, the gravitational force that acts on a boulder is many times that which acts on the pebble. Yet if you drop a boulder and a pebble at the same time, they will fall together with equal accelerations (neglecting air resistance). The principal reason the heavier boulder does not accelerate more than the pebble has to do with:

a) energy b) weight c) inertia d) surface area e) none of these

4. Analyse the motion of a small object suspended from a fixed point by a string of length l and which oscillates back and forth through a small angle θ. Taking care to *distinguish between the 'inertial' and 'gravitational' masses of the object*, what is the period of oscillation of this pendulum?

5. Possibly the first entirely *terrestrial* experimental evidence that the earth is an oblate spheroid (instead of a perfect sphere) came from pendulum observations. A pendulum adjusted so that it swings seconds at Paris, say, must be shortened before it swings seconds at the equator. Explain this effect. If a pendulum has a period of 1s at the equator, but its period at the north-pole is instead 0.997s, what is the distance from the centre of the earth to the north-pole? What is the percentage difference between the equatorial and polar radii? *[Take the earth's equatorial radius to be 6,378 km.]*

6. Consider a plane flying at a constant velocity and altitude. We have previously analysed the motion of a projectile that is dropped out of the plane in terms of Newton's dynamical laws both from the (inertial) frame of reference of the airplane itself, as well as that of a stationary (inertial) observer on the ground. As we learnt, the relation between these two frames of reference, each of which is inertial, is given by the Galilean transformations.



On the other hand, what would an observer falling alongside the projectile observe? Note that this is also the projectile's *"proper frame"* i.e. the (non-inertial) co-ordinate system attached to the freely falling object itself.

7. Given Newton's gravitational constant, $G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}$, determine the mass of:

a) the earth (assumed spherical), given that it's radius is 6,380 km.

b) the Sun given that the Earth's distance from the Sun is 1.5×10^{11} meters

8. Take the earth to be a perfect sphere with a uniform mass density. Consider a satellite orbiting the earth in a *surface-hugging* orbit (assuming of course that there are no tall mountains, trees or buildings to obstruct the path of the satellite). Then,

a) calculate the period of such an orbit;
b) show that this period does not in fact depend on the radius of the earth, but only on the earth's mass density.

9. Say you performed an experiment to measure the acceleration due to gravity in a cave deep below the surface of the earth, would the *likely* (unless you actually perform an experiment, you cannot be entirely confident of the outcome!) outcome be greater or less than at the surface – *i.e.* would objects weigh more or less in the cave than at the surface?

GROUP PROJECT – questions 10 & 11

10. Consider the hypothetical situation of drilling a shaft from a point on the earth's surface, through its centre, to the anti-podal point on the opposite side of the earth, as discussed in the lecture.

Next, a small sphere (label it A), of mass m, is held suspended at one end of the shaft and then allowed to drop into the shaft.

a) Plot its subsequent one-dimensional (in space) motion against time. What is the period of this motion?

b) Can you explain why this period is the same as the period computed in question 7a)?

c) Next, consider a second identical sphere (label it B) that is also dropped, again from rest, into the shaft but exactly 2 seconds after the first. Plot the motion of sphere B on the same graph as in part a) above. *(Assume that the two spheres do not mutually interact in any way i.e. they are simply 'transparent' to one another.)*

d) Plot the spatial separation (*i.e.* the distance) between the two spheres A & B as a function of time.

What is the maximum separation between spheres A & B, and where does this maximal separation occur?

What are the speeds of spheres A & B at this time?

e) Plot the **rate of change of the relative speed** between A & B against their **spatial separation** (*i.e.* distance between A & B).

f) Using Newton's 2^{nd} law, obtain an analytic (*i.e.* algebraic) expression for the rate of change of the relative speed between A & B. In fact, one can define the *spacetime curvature* inside the earth as:

$$\textit{spacetime curvature} = \frac{\text{rate of decrease of relative speed between A \& B}}{\text{separation between A \& B}}$$

Qualitatively, provide some justification for this formula and using it **compute the *spacetime curvature* inside the earth** in units of time (*i.e.* seconds^{-2}) and then convert your answer into units of length^{-2} (to convert, simply divide your answer by c^2).

11. **Tests of the equivalence principle & the "gravitational red-shift" effect**

The table below lists some of the seminal experiments that sought to test the validity of Einstein's equivalence principle and its prediction that for clocks in a gravitational field, a clock closer to the centre of gravitational attraction ticks more slowly than a clock that is further away, the so-called 'gravitational red-shift' effect.

Find out more information from published sources or the web about these attempts (or others you might come across) at improving the precision of experimental tests of the 'gravitational red-shift' effect. Focusing on one such attempt and based on what you have learnt, your group is to make a 15 minute powerpoint presentation during the Week 13 tutorial session. The presentation should include your understanding of the underlying physics, a clear description of the experiment and a summary of its main conclusions.

NB. A useful place to start your search for information might be: *Was Einstein Right? Putting General Relativity to the Test* by Clifford Will (this book should be available in the Scholar's Reading Room).

When	Prediction/Experiment	Result & level of precision of test
1927-60 1960-70's	**Solar gravitational "red-shift":** for wavelength of 5893 Å (angstroms = 10^{-10} m) i.e. the bright yellow emission line in solar spectrum, predicted red-shift is 0.0125 Å towards longer wavelengths	No agreement Agreement to about 5%
1960 1965	**Pound-Rebka experiment** – 'clock' was an unstable isotope Fe^{57} & comparison made between two clocks with a height difference of 74 feet, utilizing the Mossbauer effect; R.V Pound and G.A. Rebka, Physical Review Letters vol 4, 337 (1960) Improved version of experiment by Pound and J.L Snider	Agreement to about 10% Agreement to about 1%
October 1971	**J.C. Hafele and R. Keating 'Jet-lagged Clocks' experiment**	**westward flight:** predicted net gain in the 'flying clock' =275 ns (nanoseconds) of which 2/3 due to gravitational blue-shift vs. observed gain of 273 ns **eastward flight:** predicted net loss of 40 ns vs. observed loss of 59 ns; agreement to within ± 20 ns precision of this experiment
1976	R. Vessot, M. Levine et. al. hydrogen maser clock sent into suborbital flight at an altitude of 10,000 km or 1.5 r_E in a Scout D rocket. Clock uses a quantum mechanical transition between two very stable (with a lifetime of 10 million years) energy levels of a hydrogen atom that emits light with a frequency of 1,420 MHz or a wavelength of 21 cm. It's rate of ticking is compared to that of an identical clock which remains at the earth's surface	Agreement to 70 parts per million, *i.e.* 7/1000 of 1%