UNL 2201: Space, Time & Matter Tutorial 7

1. The work-energy theorem in Newtonian mechanics states that the work done on a system of particles is equal to the increase in kinetic energy of the system. Using the physics definition of work done, verify the theorem

a) for a constant force;

and

b) for an arbitrary force (not explicitly time-dependent)

that, in each case, acts on a single particle of mass m.

In special relativity however, the speed of light *c* is constant and the velocities of particles get saturated and cannot increase beyond *c*. Show that this leads to Einstein's famous equation $E=mc^2$. [*see Does the Inertia of a Body Depend Upon its Energy-Content? by Albert Einstein, Annalen der Physik, 17 (1905)*]

2. Einstein's original approach was to argue that no material object can be made to reach a velocity v equal or greater than the speed of light *c*, with respect to the observer. Show that the relativistic definition of kinetic energy is in conformity with this. What must the *proper mass* of an object be in order that it moves at a speed *c*? [*see The Electrodynamics of Moving Bodies by Albert Einstein, Annalen der Physik, 17 (1905)*]

3. Calculate the mass of an electron when it has a speed of (a) 4.00×10^7 m/s in the CRT of a television set, and (b) 0.98c in an accelerator used for cancer therapy.

4. In special relativity, we have the <u>mass equivalence of energy</u>. Nature generally exhibits *symmetry* in its operations and one may therefore suspect that a particle of matter might lose some of its mass by giving up a corresponding amount of energy (for example, by slowing down a particle moving at high speed). However, there is a more striking instance of this exchange between mass and energy: it is possible for <u>all</u> of a particle's mass to be transformed into energy!

One of the elementary particles that exists in nature is the charge neutral pion or π° meson. Say such a π° meson (rest mass, $m_o = 2.4 \times 10^{-28}$ kg) travels at a speed $v = 0.80c = 2.4 \times 10^8$m/s; (a) what is its kinetic energy? Compare this to a classical calculation. (b) How much energy would be released if the π° is transformed completely into electromagnetic radiation?

5. The equation $\Delta m = E/c^2$ tells us how much mass loss Δm must be suffered by a nuclear reactor in order to generate a given amount of energy E. Which of the following statements is correct?

(a) The same equation also tells us how much mass loss Δm must be suffered by a flashlight battery when the flashlight puts out a given amount of energy, E.

(b) The equation applies only to nuclear energy in a reactor, but not to chemical energy in a battery.

6. Consider a motorcycle powered with super-powerful electric batteries, and an electrically powered MRT train that are each driven to speeds approaching the speed of light. Measurements of each from the point of view of an observer at rest on the ground will indicate an increase in the mass of the

(a) motorcycle (b) MRT train (c) both (d) neither.

7. Energy release in a *'fission'* or atom bomb.

(a) How much energy is released in the explosion of a fission bomb containing 3.0 kg of fissionable material (eg Uranium 235 or Plutonium 239)? Assume that 0.10% of the mass is converted into released energy.

(b) What mass of TNT would have to explode to provide the same energy release? Assume that each mole of TNT liberates 3.4 MJ (Mega joules) of energy upon exploding. The molecular mass of TNT is 0.227kg/mol.

(c) For the same mass of explosive, how many times more effective are atom bombs compared to TNT explosions? [*ie compare the fractions of the mass that are converted to energy in each case.*]

8. Derive, starting from Einstein's relation $E=mc^2$, the following equation:

$$E^2 = p^2c^2 + m_0^2 c^4$$

where E is the relativistic energy, p the relativistic momentum, m_0 the 'rest' or proper mass and c the speed of light. *Hint: use the relationship between the relativistic and rest masses of a particle* $m = m_{\mathbf{0}}\sqrt{(1 - v^2/c^2)}$.

What can you deduce from this equation about a particle that is at rest (as measured in its proper or rest frame)? What can you say about a particle that happens to have vanishing rest mass $m_{\mathbf{0}}$?

9. A defining property of a vector in two (or three) dimensional space is that its length remains unchanged or is said to be 'invariant' under rotations of the co-ordinate axes in two (or three dimensions) as well as under Galilean transformations between inertial frames. In special relativity, *spacetime* is four dimensional and lengths of ordinary vectors or spatial distances are no longer invariant between inertial frames. Instead, it is the square of the *spacetime* interval connecting two given events, for example (ct, x, y, z) and (0, 0, 0, 0) that is invariant under the Lorentz transformations.

Analogously, consider the following four dimensional 'vector': (E; cp_x, cp_y, cp_z) where E is the energy and (p_x, p_y, p_z) are the spatial components of momentum. Then the Lorentz transformations between two inertial frames moving with a relative velocity v (= v_x , *i.e.* the relative motion is only in the x-direction) that relate the energy and momentum as measured in each frame respectively are as follows:

$$E' = [E - vp_x] / \sqrt{(1 - v^2/c^2)}$$

$$p_x' = [p_x - vE/c^2] / \sqrt{(1 - v^2/c^2)}$$

$$p_y' = p_y$$

$$p_z' = p_z$$

For simplicity, set $p_x' = p'$ and $p_x = p$ and assume also that the momentum components p_y and p_z are both zero.

Try to construct a combination of E and p (refer to the previous question) that is invariant under the transformations above. What can you conclude from the invariant you have formed?