UNL 2201: Space, Time & Matter — Tutorial 4

1. Suppose you are going for a long bicycle ride. You cycle one hour at 5 kilometres per hour. Then three hours at 4 kilometres per hour and then two hours at seven kilometres per hour. How many kilometres did you ride?

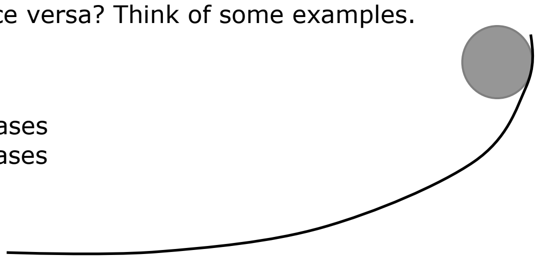
a) five b) twelve c) fourteen d) thirty-one e) thirty-six

2. In the old Greek fable of Achilles and the tortoise, they have a race with Achilles (since he runs much faster, say at 15 miles per hour) giving the tortoise a lead of 22 feet. For simplicity, let us assume that the tortoise is so slow that in effect he does not move at all!

Now, if space is assumed to be continuous, it must be infinitely divisible and therefore the distance between Achilles and the tortoise at the start of the race may be divided up into an infinite number of tiny segments, all of which have to be crossed by Achilles if he is to catch up with the tortoise. But one can never traverse an infinite number of segments in any finite amount of time and Achilles will never catch up with the tortoise – or at least so argued Zeno who framed this famous 'paradox' that is named after him. Zeno concluded that all motion is therefore impossible! What was the fallacy in Zeno's reasoning?

3. Aristotle's mechanics was based on *velocity*, whereas Galileo's was based on the concept of *acceleration*, or the rate at which *velocity* changes. If the *velocity* of an object is zero, does it mean that the *acceleration* is zero? What about vice versa? Think of some examples.

As the ball rolls down this hill



a) its speed increases and acceleration decreases
b) its speed decreases and acceleration increases
c) both increase
d) both remain constant
e) both decrease

4. A motor bike starts from rest and uniformly accelerates to 60 km/h in 10 seconds. Selecting suitable variables, sketch a graph that describes this motion and use your graph to figure out how far the motor bike has travelled after 10 seconds? How far would it have travelled (i) during the first second, and (ii) in each successive second after the first?

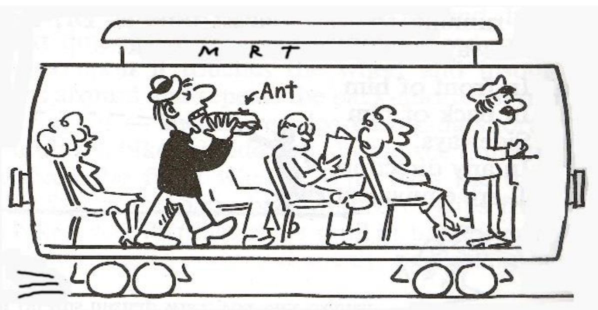
5. A sled with a mass of 1 kg is set in motion over frictionless ice by a miniature rocket motor. After the rocket fuel is expended, the sled coasts along over the ice surface at 1 m/sec. How much force did the rocket exert on the sled?

a) 1 Newton b) less than 9.8 Newtons c) more than 9.8 Newtons

d) There is no way to tell from the information given.

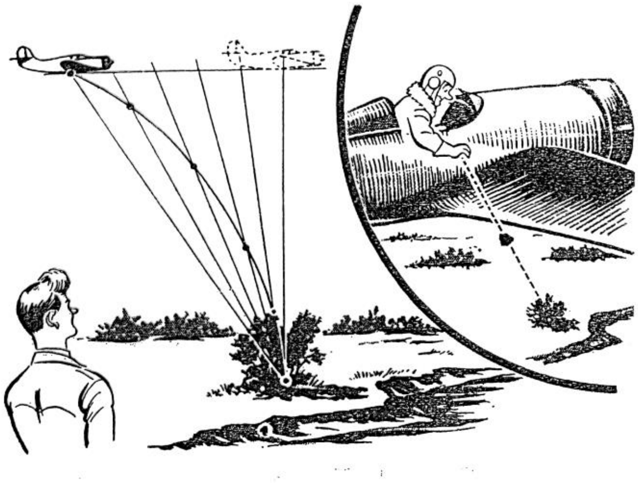
6. Consider an object of mass m, suspended at a certain height above the Earth's surface; since it is stationary it has zero momentum. However, once it begins to fall, it has speed and hence non-zero momentum. Does this mean that the *Law of Conservation of Momentum* is violated by the falling object?

7. A MRT carriage is approaching the Somerset station at 1m/s. A person in the carriage is facing forward and walks forward with a speed of 20 cm/s relative to the seats in the carriage. The person also happens to be eating a hot-dog which is entering his mouth at the rate of 4 cm/s.



An ant on the hot-dog is running to the end of the hot-dog, away from the person's mouth. The distance between the ant and the end of the hot-dog towards which it is running, is diminishing at the rate of 2cm/s. Can you figure out how fast the ant is approaching Somerset station?

8. Consider a plane flying at a constant velocity and altitude. Analyse the motion of a projectile that is dropped out of the plane from the frame of reference of the airplane itself.



What do you understand by Galilean Relativity and explain how the motion of airplane and projectile illustrates this principle.

Use the Galilean transformations to obtain the trajectory of the projectile with respect to the Earth's frame (neglect the rotation of the Earth about its axis).

9. An airplane flies horizontally with a constant velocity of 600 km/h, at a height of 2km. Directly over a marker it releases an empty fuel tank. Neglecting air resistance, how far ahead of the marker does the tank hit the ground? At this instant of time, is the airplane ahead or behind the tank?

10. Indicate on a diagram, the sum and difference of two vectors $\mathbf{v}_\mathbf{A}$ and $\mathbf{v}_\mathbf{B}$ and use this to analyse the following situation: two trains A and B are traveling in opposite directions along straight parallel tracks at the same speed v = 60 km/h. A light airplane crosses above them. A person on train A sees it cross at right angles, while a person on train B sees it cross the track at an angle θ_B = 30°.

(i) At what angle θ_{ground} does the airplane cross the track as seen by an stationary observer on the ground?

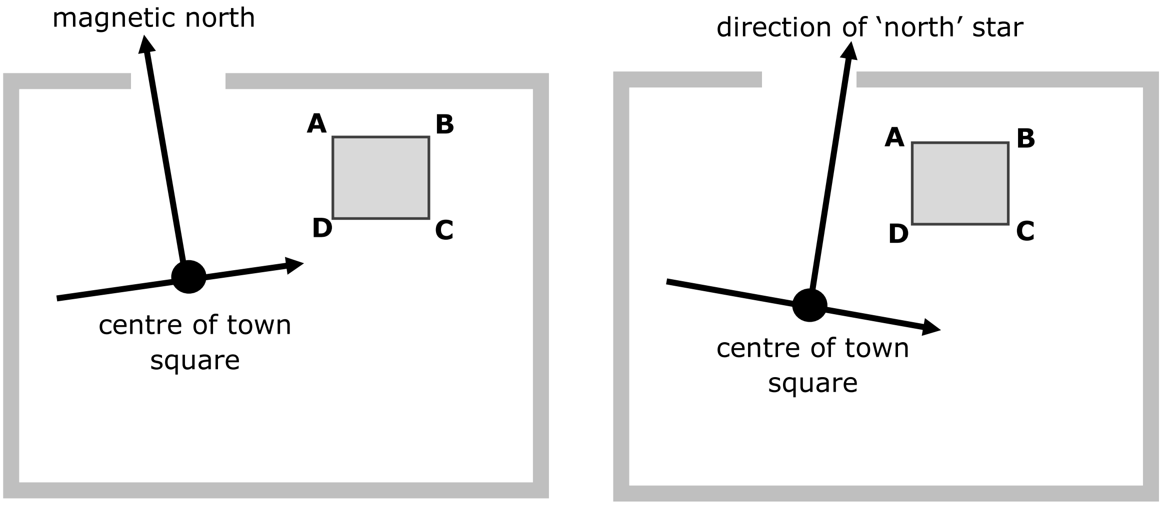
(ii) What is the airplane's speed relative to the ground, v_{ground} ?

11. Newton's mechanics is governed by his second law **F** = m**a** or in components,

F_x= mdx^2/dt^2, with similar equations in the y and z directions. To check whether this law is true, we would need to choose a co-ordinate system so as to be able to measure x, y and z in three perpendicular directions as well as the forces along these directions. These must each be measured from some point (the origin of the chosen co-ordinate system). Show that it does not matter where we take this origin to be, or in other words, that Newton's second law is *symmetric or invariant under spatial translations.*

12. Show that it also does not matter in which direction the axes of the co-ordinate system are chosen; that is to say, Newton's second law is *symmetric or invariant under spatial rotations.*

13. Imagine that there are two groups of surveyors each carrying out a land survey within the same walled town (assumed to be flat). One group works exclusively in the daytime and identifies due north using a magnetic compass, while the other group works only during the night and identifies due north using the 'north' star (Polaris). Each group has been tasked with marking out the same rectangular plot of land ABCD within the town.



The problem is that, while agreeing on the location of the centre of the town, the two groups of surveyors cannot seem to agree about the location of the corners of the plot of land (as you can see from the table below).

	Day time surveyor's axes oriented to magnetic north		Night time surveyor's axes oriented to 'north' star	
	Eastward (meters)	Northward (meters)	Eastward (meters)	Northward (meters)
Centre of town square	0	0	0	0
A	401	295	395	303
B	501	294	495	304
C	500	236	496	204
D	400	194	396	203

You are asked to resolve the problem – what would you do to obtain the locations of the corners of the plot of land in such a way that both groups of surveyors are in agreement?

14. How would Newton's second law change from the point of view of an observer who is in uniform motion?

What happens instead from the point of view of an observer who is

a) Undergoing uniform (*i.e*. constant) acceleration in a straight line

b) Undergoing uniform circular motion (*i.e*. moving in a circle at a constant speed)?

15. Is the room we are in really an *inertial* frame of reference? Explain your answer considering that the Earth spins around its own axis once every day and revolves around the Sun once a year.

In physics it is interesting and often instructive to obtain the same result in more than one way. For example, the Law of Conservation of Momentum can be derived from (or is equivalent to) Newton's Third Law. It is also possible to obtain or to understand the Conservation of Momentum using instead the Principle of Galilean Relativity (as the following problems indicate). **The principle states that** ***any mechanical experiment will have exactly the same outcome in a system in uniform motion that it had with the system at rest*****.**

16. Utilising the Principle of Galilean Relativity, deduce Newton's First Law, sometimes referred to as the *Principle of Inertia – in the absence of any external force, a uniformly moving body continues to move uniformly* – from the simpler and perhaps intuitively more plausible statement, namely that *in the absence of an external force, a body at rest remains at rest.* What do you conclude from this?

17. Suppose that two identical, perfectly elastic, billiard balls move towards one another at the same speed and collide head-on. (Perfectly elastic implies that the kinetic energy in such a collision is conserved.) After the collision, both balls are observed to recoil with their original speed (simply by using the symmetry of the situation instead of momentum conservation, you could argue that they must recoil with equal speeds).

What happens if one of the balls is at rest and you shoot the other one towards it at 4 m s^{-1}?

18. Two identical sticky billiard balls have the property that if they are fired at each other with equal speeds, they stick together upon colliding and the resulting compound ball is stationary. If such a ball is fired at 4 m s^{-1} directly at another identical sticky ball and the two stick together, with what speed and in what direction will the compound ball move after the collision?

19. Imagine we have two perfectly elastic balls, one very large and the other very small. If the big ball is stationary and the small one is fired directly at it, the small ball simply bounces back in the direction it came from with the same speed, while the big ball remains at rest. (Think of throwing a tennis ball directly at a bowling ball for instance.)

With what speed will each ball move after the collision, if instead the small ball is stationary while the big ball rolls towards it at a speed of 4 m s^{-1}?

What happens if the big and little ball approach one another with the same speed, say 2 m s^{-1}?