UNL 2201: Space, Time & Matter **Tutorial 2**

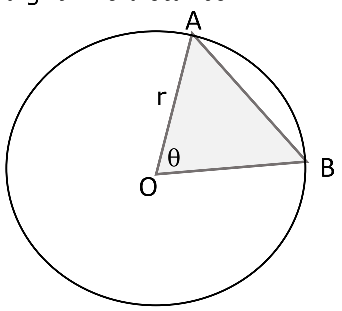
1. You measure the elongation of Venus on a series of nights around its greatest elongation. From your measurements you decide that the greatest elongation of Venus is 46° 18´. How far is Venus from the Sun?

2. An observatory sends a radio signal to Mercury. Exactly 21.1108 minutes later it detects the echo. How far away is Mercury? *Take the speed of light to be 2.99792458 x 10^5 kms^{-1}.*

3. a) One radian is the angle subtended at the centre of a circle by an arc equal to the radius of that circle. Convert 1 arc second (or 1") into radian measure. Conversely, how many arc seconds is one radian?

b) For sufficiently small angles, Sin θ ≈ tan θ ≈ θ (where θ is measured in radians); either prove this, or formulate an argument which explains why this result must be true.

c) Consider any two points A and B on the circumference of a circle, centre O and radius r. The 'distance' AB is usually taken to mean the length of the straight-line joining A and B. Instead, let's say we choose to measure the separation between A and B along the arc subtended by an angle θ radians, namely r θ which in general could be very different from the straight-line distance AB.



What if we choose A and B to be two *nearby* points? In fact, the idea of nearby points is, mathematically speaking, not a trivial one. To state it properly leads us to the concept of a change in coordinates from point to nearby point, a change so small that it counts as "differential", a concept conventionally abbreviated as d(distance) or d(angle) in the present context and we write

d(distance) = r x d(angle)

It is clearly wrong to apply this formula to obtain the 'distance' between A and B when the angle θ is large. For example, if A and B are points chosen to be diametrically opposite, then the straight line 'distance' AB would be 2r, whereas the 'distance' measured along the circular arc using the formula above would instead be πr – an error of about 57%!

However, what if the angle between A and B is very small? Compute the percentage error when θ is say, 0.1 radians.

4. Calculate the distance – in both astronomical units and *parsecs* – of *α Centauri* (parallax 0.742′′), *61 Cygni* (parallax 0.287′′) and *Vega* (parallax 0.125′′).

5. How would our parallax measurements change if we could make our observations from the surface of Mars? How large would the *parsec* be for Martian astronomers?

6. One of the first reasonably accurate estimates of the distance to the stars was made by Isaac Newton who estimated the distance to Sirius. Newton began by making the usual assumption that all stars, including the Sun, emit the same amount of light i.e. they all have the <u>same</u> *luminosity*. He further assumed that there was no loss of light on the way to the Earth. The observed apparent brightness of Sirius as compared to the Sun would thus provide a measure of the relative distance to Sirius. The novel feature of Newton's argument was that, by observation, he noted that Sirius appeared to be about as bright as the planet Saturn. According to Newton, the apparent angular radius of Saturn as seen from the Sun is 9′′. He used these observations to obtain an estimate of about 800,000 AU for the distance to Sirius. [*The modern value for the distance to Sirius is 546,960 AU.*] How did Newton obtain his result?

7. A star varies in apparent magnitude from +7.1 to +7.8. Find the relative increase in brightness from minimum to maximum.

8. How many *sixth-magnitude* stars equal the brightness of a single *first-magnitude* star?

9. What does one mean by the *luminosity* of a star? How would you decide which is more luminous – the Sun or Sirius – and calculate the ratio of their respective luminosities.

10. *61 Cygni* has an *apparent magnitude* of 5.2 and an *absolute magnitude* of 7.49. How far away is *61 Cygni*? Compare this distance with the value obtained for *61 Cygni* in question 4.

11. Write an expression for the distance to a star in terms of its *distance modulus* μ_0.

a) What is the distance modulus for the Sun

b) Which star, other than the Sun, has the smallest distance modulus and what is μ_0 for this star?

12. If we make a 0.01 magnitude error in measuring the apparent magnitude of a star, what percentage error does this introduce in our distance estimate for that star? *Assume that we know the absolute magnitude of the star exactly.* What if the magnitude error is 0.1? What if the magnitude error is instead 1.0?