Tutorial 5

UNL 2201: Space, Time & Matter

1) The following experiment was performed in 1851 (more than 50 years before Einstein published his 1905 paper) by Fizeau to measure the speed of light in water moving within a tube.



The speed of light in *stationary* water is known to be *less* than its speed in empty space and is conventionally written as c/n where c is the speed of light in vacuum and n is the *refractive index* of water.



From the "non-relativistic" or Galilean velocity addition law, one would expect that if water flowed through a pipe with velocity v, then the speed of light in the moving water would be its speed in stationary water increased by the speed of the water flowing in the pipe: w = c/n + v. However, what Fizeau observed instead was:

$$w = c/n + v (1 - 1/n^2)$$

Show that this result can be easily understood as an elementary consequence of the "*relativistic*" addition of velocities law.

(NB. At the time, Fizeau's experimental result was viewed as confirmation of a rather elaborate contemporary calculation based on the idea that the water was partially successful in dragging the ether along with it.)

2) Which of the following facts, if definitely established experimentally, would violate the *Special Theory of Relativity*?

- (a) Objects can travel faster than the speed of light
- (b) Nothing can go faster than the speed of light
- (c) If an object is traveling faster than the speed of light, it immediately slows to a speed less than that of light.

3) There are many different kinds of clocks: hour glasses, mechanical clocks, quartz clocks, atomic clocks, light clocks and biological clocks (your heart beat for example). We have seen that motion appears to slow down one particular type of clock viz. a light clock; does it necessarily follow that all other kinds of clocks must be affected equally? If so, why?

4) You are holding a long pole parallel to the ground and at its mid-point when you drop it. As you perceive it, the ends of the pole, A and B, strike the ground simultaneously and hence you



think that the pole has fallen horizontally. But your friend who is dashing past you in the direction AB *at nearly the speed of light*, perceives end B strike the ground before end A and therefore thinks the pole is tilted to the right as it fell. Is this true or false?

5) Analyse the *relativity of simultaneity* 'thought experiment' discussed in the lecture from O_1 's point of view. What conclusion are you able to draw from this?

6) A car traveling 100 km/h covers a certain distance in 10.00 s according to the driver's watch. What does an observer stationary with respect to the earth measure for the time interval?

7) At what speed do the *relativistic* formulas for lengths and time intervals differ from classical or *non-relativistic* values by 1.00%? (*This is a reasonable way to estimate when to do relativistic calculations rather than classical ones.*)

8) Experimental tests of the *time dilation* effect in special relativity are by now routine, and can be carried out with an extremely high degree of precision (typically at particle accelerator facilities such as CERN in Switzerland at which particles can be accelerated to almost the speed of light).

However, one of the earlier attempts to verify this prediction of special relativity employed the radioactive decays of particles created by cosmic ray collisions in the upper atmosphere. One such particle is known as the *muon*, similar to an electron but 200 times more massive. Muon are unstable and undergo radioactive decay with a half-life of 2.2×10^{-6} s, as measured in the muon *rest-frame*.

Muons, created in the upper atmosphere, can have speeds approaching 0.999c; assuming Galilean relativity to be correct, how far would such muons travel on average?

Calculate instead, the half-life of such muons according to special relativity in the *earth's frame* and, consequently, the distance such muons would be able to travel through the atmosphere.

In 1963, Frisch and Smith carried out an experiment at Mount Washington in New Hampshire, USA. At the top of the mountain, they observed approximately 563 muons per hour with the average measured speed of 0.995c. However, at the base, some 1,907 meters below the summit, they detected 412 muons per hour.

Assuming a mean lifetime of 2.2 $\mu s,$ only 27 muons would reach this location if there were no time dilation.



Frisch, D. H.; Smith, J. H. (1963). "Measurement Microseconds of the Relativistic Time Dilation Using μ-Mesons". *American Journal of Physics*. **31** (5): 342–355. 9) The rest radius of the Earth is 6370km, and its orbital speed about the Sun is 30km/s. Suppose Earth moves past an observer (who is stationary with respect to the Sun) at this speed. To this observer, by how much would the Earth's diameter be contracted along the direction of motion?

10) Suppose you decide to travel to a star 90 light-years away. How fast would you have to travel so that the distance would only be 25 light-years?

11) A certain star is 75.0 light-years away. How long would it take a spacecraft traveling 0.950*c* to reach that star from earth, as measured by observers:

a) on earth,

b) on the spacecraft?

c) What is the distance travelled according to observers on the spacecraft?

d) What will the astronauts making the journey compute their speed to be from the results of b) and c)?

12) A spear 10 m long is thrown at a relativistic speed through a pipe that is 10 m long. Both these dimensions are measured when each are at rest. When the spear passes through the pipe, which of the following statements best describes what is observed?



a) The spear shrinks so that the pipe completely covers it at some point.

b) The pipe shrinks so that the spear extends from both ends at some point.

c) Both shrink equally so that the pipe just covers the spear at some point.

d) Any of these, depending on the motion of the observer

13) You are in a spacecraft that happens to be 1 km long. An alien spacecraft flies past you. When the passing alien is alongside you, you note that the alien craft is exactly as long as your own craft. That is, the alien craft nose is abreast of your tail when the alien craft tail is abreast of your nose – according to your time. Answer the following questions and provide brief justifications for your answers.

- a) Are the two craft identical in length?
- b) Does the alien measure its craft to be as long as, longer or shorter than yours?
- c) Since you measure the alien craft nose, which is abreast of your tail, *at the same time* the alien tail is abreast of your nose, why doesn't the alien observe the same coincidence of events?