UNL2206, Nature's Threads: Tutorial 4

1) A small test charge -q is placed at a certain distance from the centre of a spherically symmetric cluster of identical charges, each of +Q. Coulomb's law and the principle of superposition tell us that the test charge will be attracted towards the cluster by some electrostatic net force. Now consider the situation whereby neither the test charge nor the centre of the cluster moves, but the cluster uniformly expands. As a result of this expansion, some parts of the cluster will be closer to the test charge, but other parts will be further away. After the expansion, the electrostatic forces of the cluster on the test charge will



2) A thin, spherical shell of radius r_0 and made out of conducting material possesses a total net charge of Q that is uniformly distributed on it. Determine the electric field at points (a) outside the shell, and (b) inside the shell.

3) Consider a sphere (not necessarily a conductor) which has a uniform electric charge distribution throughout its volume. Determine the electric field (a) within the sphere, and (b) at any point outside.

4) A very long, straight wire possesses a uniform charge per unit length. Calculate the electric field at points (a) within the wire, and (b) outside the wire but far from its ends.

5) An infinite, thin plane sheet has a uniform charge distribution, say σ Coulombs per unit area. Calculate the electric field on each side of the sheet.

6) Using Gauss' law, show that the charge density in the *interior* of any conductor in equilibrium must be zero. What if the conductor has an empty cavity of *arbitrary* shape hollowed out inside it - can there be any non-zero charge distribution on the inner surface of the cavity, or any electric fields within the cavity?

7) Show that the electric field just outside the surface of any good conductor of arbitrary shape in the immediate vicinity of a given point on the conductor's surface is given by $E = \sigma / \epsilon_0$ where σ is the surface charge density (in general non-uniform) on the conductor at that point.

Account for the difference between this result and that of question 5) i.e. why is the electric field in that case only $E = \sigma / 2\varepsilon_0$?

8) A *capacitor* consists of two parallel, conducting plates, each of area A and a distance d apart. The plates are charged with a value +Q and -Q respectively.

- a) Determine the electric field outside the two plates.
- b) Determine the electric field between the two plates.
- c) What is the potential difference across the two plates?
- d) What is its *capacitance* (the capacity for the plates to hold electric charge)?

9) It was Faraday who first observed experimentally that if an insulating material or *dielectric* is inserted between two parallel, conducting plates, the *capacitance* <u>increases</u>. What can you conclude about the electric field between the plates in the presence of a dielectric and try to explain the increase in *capacitance* in terms of a simple model for a *dielectric*.

- 10) Answer the following and suggest some examples:
 - a) If an object has a non-zero *interior* (ie bulk or volume) charge distribution and is in equilibrium, can the object be a conductor?
 - b) If an object has a non-zero bulk or volume charge distribution and is not in equilibrium, <u>must</u> the object be a conductor?
 - c) If an object has a non-zero *surface* charge distribution, must the object be a conductor in equilibrium?

11) A hollow metal box is placed between two parallel charged plates as illustrated below. Sketch the electric field both inside and outside the box. Justify your result using Gauss' Law.



12) If you are inside a hollow conductor, you are completely shielded electrically from any outside charges and such a set-up is sometimes referred to as a *Faraday cage*. But what if you reverse the situation by placing a metal shield around a charge? – by Gauss' s law, you should detect an electric field from the charge inside the metal shield. Is there any way to prevent the electric field with the charge as it's source from reaching you outside the shield?

13) Consider a Gaussian surface S that encloses a cube of side a with one corner at the origin and the diagonally opposite corner at (a, a, a). It is in a uniform or constant electric field directed along the x-axis. Find the flux through S and the charge enclosed.

Group Assignment - Faraday's ice pail experiments

For the following, again form groups of students. Each group should assemble the home-made devices outlined, perform the corresponding 'experiments' described below (and any further ones they can think of) noting carefully the various outcomes. Bring your working devices to class; groups will be expected to report on their collective results to the tutorial class.

Consider an electroscope or charge electrometer connected to the exterior of a conductor (ice pail) into which objects can be placed. Faraday hung an electrically charged object A, with charge +Q, by an insulating string. On lowering A into the uncharged ice pail, without A touching the sides, the electrometer reading increased, but once A was about 10 cm below the top of the pail, the electrometer reading stabilized, even if A was moved about. When A was lifted back out of the ice pail, the electrometer reading returned to zero. Moreover, when A was within the ice pail, the electrometer reading was the same as if all the charge on A had been placed on the ice pail.

This indicated that (1) if the charged object A is far enough within the ice pail, then a charge distribution goes to the outer surface that is the same as if +Q had been placed directly on the ice pail; (2) the charge +Q on A and the charge -Q remaining on the inner surface together produce electric fields that have no effect on the charge distribution on the outer surface of the ice pail.

Next, on lowering a charged *conducting* object A into the ice pail, the electrometer response increased, as before. Touching A to the inside of the pail, temporarily making a pail-object combination, yielded no change in the electrometer reading. However, lifting out A, the electrometer reading remained at the *same* value as when A was within the ice pail. Moreover, placing A within another ice pail connected to an electrometer gave no response. This indicated that, while A and ice pail were in contact, (1) there was no charge on the inner surface of the ice pail-object combination; (2) there was no charge in the volume associated with A, thought of as part of the ice pail-object combination. Hence, touching the charged conducting object to the interior of the ice pail made A transfer its charge to the ice pail.

Although A originally was attracted to both the interior and the exterior of the neutral ice pail, after contact with the interior it was attracted only to the exterior of the now charged ice pail. This behaviour after contact with the interior can be explained using electrostatic induction if now A is uncharged, and the ice pail is charged on the outside but not on the inside.

- A. Try to interpret the above in terms of field lines.
- B. Reproduce Faraday's results for yourself using the following: 1) a charged plastic comb as the charge source A; 2) a small tin food can or soft drinks can with its top removed and attaching to it insulating handles of folded over sticky tape; 3) a large food (eg baby milk formula) can with its top removed which serves as the ice pail; and 4) a versorium and/or an electroscope. The larger can should be mounted on an insulating surface such as a styrofoam cup. The versorium should be placed near an edge of the large can, where the electric field will be largest.



The comb can be charged and used as an insulating charge source within the can. Placing the charged-up comb within the small can, and then touching the small can charges the small can, which can be used as a conducting charge source within the large can. The small can and comb are oppositely charged, as shown by the lack of versorium response when both are within the larger can.